



# Study of cross-spectra of velocity components and temperature series in a nocturnal boundary layer

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## 1. Introduction.

The main characteristic of the Planetary Boundary Layer (PBL) is the turbulent flow that can be understood as the motions of many superimposed eddies with different scales, which are very irregular and produce mixing among the different atmospheric properties.

Spectral analysis is a widely statistical tool used to know the size of eddies into the flow. The Turbulent Kinetic Energy is split in fractions for each eddy scale by mean of the power spectrum of the wind velocity components. Also, the fluctuation of the other variables as temperature, humidity or material particles present in the atmosphere can be divided according to the importance of the different scales in a similar way than the wind.

A Cross-spectrum between two time series is used in meteorology to know their correlation in the frequency space. Cospectrum, or real part of the cross-spectrum, amplitude and, specially, coherence give us many information about the low or high correlation between two variables in a particular frequency or scale (Stull, 1988).

Moreover, a MultiResolution Flux Decomposition (MRFD) is used to identify different scales (some of them larger than turbulence) into the atmospheric flow. It is a multivariate and multiscale spectral tool for computing turbulent fluxes and studying the contribution by different timescales  $\tau$  (Howell & Mahrt (1997) )

## 2. Method

The spectra and cross-spectra were performed from the Blackman-Tukey method, widely used in the time series studies (Blackman & Tukey, 1958) and, where it is needed the correlation function of the time series analysed.

### • Cross-spectrum.

Let be two discrete time series  $A(t)$  and  $B(t)$  and its corresponding forward Fourier Transforms  $F_A(n)$ ,  $F_B(n)$  that are both complex functions of frequency  $n$ .  $G_A = |F_A(n)|^2$ ,  $G_B = |F_B(n)|^2$  are the unfolded spectral energy for  $A$  and  $B$  respectively (Stull, 1988). It is define the cross spectrum between  $A$  and  $B$  by

$$G_{AB} = F_A^* \cdot F_B = Co - i Q \quad (1)$$

where

$$Co = F_{Ar} F_{Br} + F_{Ai} F_{Bi} \quad (2) \text{ (cospectrum)}$$

$$Q = F_{Ar} F_{Bi} - F_{Ai} F_{Br} \quad (3) \text{ (Quadrature spectrum)}$$

As  $F_A$  and  $F_B$  are functions of the frequency  $n$ , cospectrum and quadrature spectrum are also function of  $n$ :  $Co(n)$  and  $Q(n)$ .

From cross spectrum it is define the Amplitude Spectrum,

$$Am = \sqrt{Co^2 + Q^2} \quad (4)$$

Coherence spectrum  $Coh$  (or coherence function in a continuous form)

$$(5)$$

and the phase spectrum  $\phi$

$$\tan \phi = Q / Co \quad (6)$$

Coherence spectrum is a real number in the range 0 to 1. It acts very much like a frequency dependent correlation coefficient.

In the work, we used  $Coh$  to find a possible correlation between too variables.

### • Multiresolution Flux Decomposition (MRFD)

• It is a multivariate and multiscale spectral tool for computing turbulent fluxes and studying the contribution by different timescales  $\tau$  (Howell & Mahrt, 1997)

• In stable conditions, MR cospectra of vertical heat flux  $\langle w\theta \rangle$  is usually studied.

• The effect of meso-scale events ( waves, irruption of katabatic flows, etc. ) in the spectra of turbulent motions can be tracked by studying the change in the contribution from different timescales, and the evolution of parameters related to the shape of the cospectra, such as  $\tau_{MAX}$  or  $\tau_{GAP}$  (see adjacent figure).

## 3. Data.

Dataset has been obtained from a 10 m height mast (Fig. 1) installed in the CIBA, Research Centre for the Lower Atmosphere, located in Valladolid province (Spain), which is on a quite flat terrain (Cuxart et al., 2000; Viana et al., 2009).

The instrumentation available for this study is the following:

Levels 1.5, 3, 5, 7.5 and 10 m: cup anemometers, vanes, thermometers

Levels 1.5 and 10 m: sonic anemometers, humidity.

The analysis performed belong to a Summer 2009 week, from 24<sup>th</sup> to 31<sup>st</sup> July (Fig. 2).

Sonic anemometers have a sampling rate of 20Hz, give the three velocity components and the temperature from the velocity of the sound (sonic temperature).

Cross-spectra and analysis of MRFD was obtained from the sonic anemometers. Moreover, wind and stratification conditions, as well as others complementary properties were got by the reminder instruments.



Figure 1: Picture of 10 m tower used in CIBA. 5 levels: cup anemometers, vanes and thermometers; at 1.5m and 10 m height humidity and sonic anemometers. Terrain is very flat without obstacles nearby.

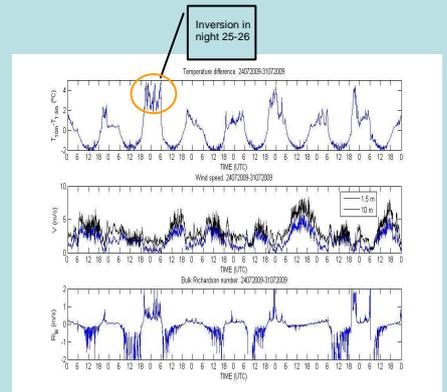
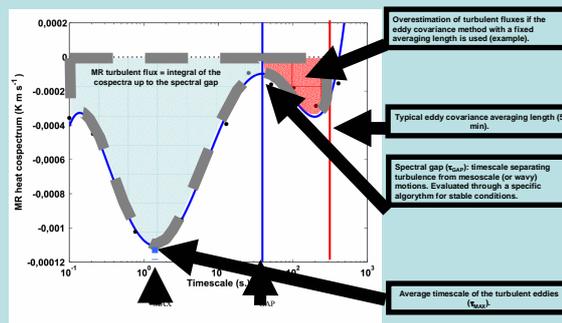
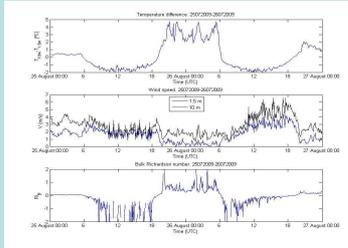


Figure 2: Complete week of study from July 24<sup>th</sup> to 31<sup>st</sup>. In the upper panel it is shown the temperature difference between 10m and 1.5m. Intermediate panel draws wind module for the two levels studied, and lower is Bulk Richardson number obtained from the 2 extreme levels of the tower.

## 4. Results and discussion.

Figure 2, shows  $\Delta T_{10-1.5m}$ ,  $V$  and  $Ri_B$ . The 25-26 night present a strong surface inversion, with slow winds and certain wind shear between 1.5 and 10m;  $Ri_B$  is positive and often supercritical along the night. However, positive difference of temperature ( $T_{10m} - T_{1.5m}$ ) presents some variability. Figure 3 draws days 25-26 July in more detail.





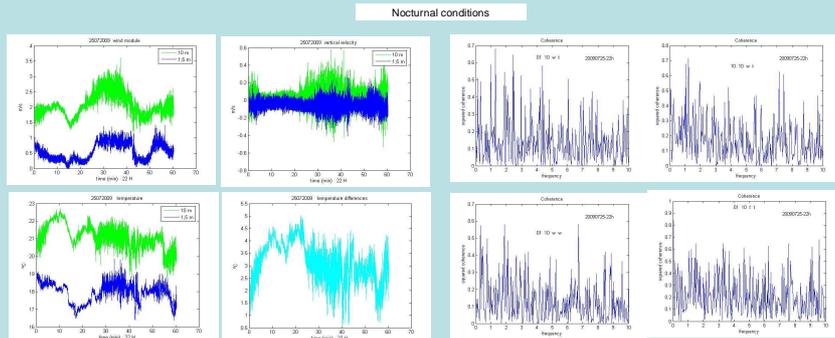
**Figure 3 :** Zoom of the previous figure in order to stand out the strong of inversion at 25-26 night and the shear close to surface. In the lower panel Bulk Ri number gives the kind of stratification as combination of temperature gradient and wind shear.

Cross-spectra between variables of the two levels, specially, wind vertical component and sonic temperature, under stable stratification are studied, in order to improve the knowledge of the properties of the momentum and heat fluxes near the ground in the PBL.

Next it is presented the time series of the studied variables for 1 hour record in nocturnal condition (Fig. 4) and near to the evening transition (Fig. 6). There is a slow wind (2 m/s) at night and some higher (6 m/s) at sunset. The vertical velocity shows a large variability, especially in the afternoon. Temperatures are characterized by a negative trend in transition.

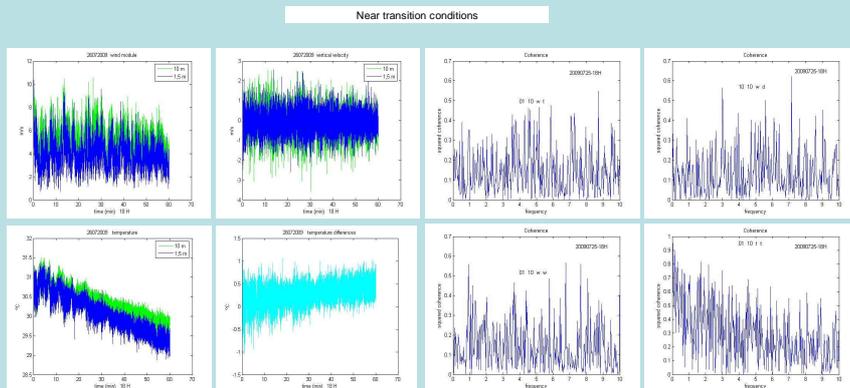
The coherent spectra (Figs. 5 & 7) show low correlation among different properties except for temperatures in the near transition time between the two levels in the longest scales likely as a consequence of the trend and with the presence of turbulent large scales detected by the MRFD method (Fig 8).

Figure 8, presents the result of the MultiResolution flux decomposition for TKE and  $\langle wt \rangle$  (heat flux) along the 6 last hours of the 25<sup>th</sup> July. It can be seen that before sunset (19:30 UTC approx.) the convective turbulence has temporal scales between 10 and 10<sup>3</sup> s. Along this stable night small high frequency turbulence is present, while the presence of gravity waves can generate larger scales contributions and counter-gradients heat fluxes.



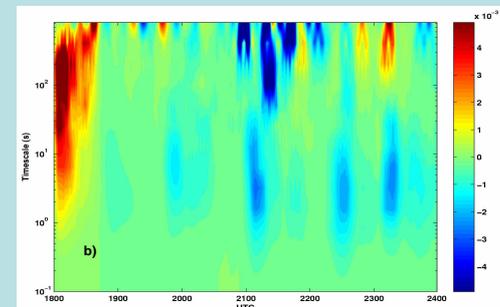
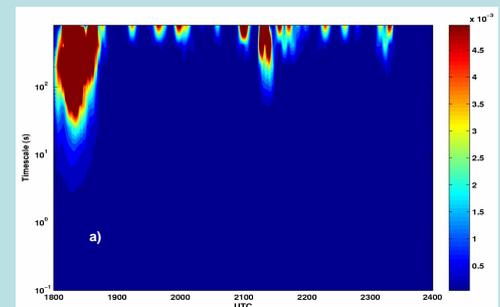
**Figure 4:** One hour time series of the variables: wind speed, vertical velocity and temperature at two levels, and temperature differences between them. The initial time is 22:00 UTC. It can be seen a strong inversion around 3 °C.

**Figure 5:** Coherence function between several variables indicates a low correlation for all the frequencies, except for some individual, mainly between vertical velocity ( $w$ ) and temperature ( $t$ ), independent of the levels used.



**Figure 6:** As fig.4 for 18:00 UTC. In this case temperature presents a clear trend as corresponding to the beginning of the evening transition.

**Figure 7:** Coherence function is lower for all the correlations except for temperature between 1.5 m and 10 m levels (low-right in the panel).



**Figure 8:** MRFD, of (a) Turbulent Kinetic Energy TKE and (b)  $\langle wt \rangle$  heat flux from 18:00 to 24:00 UTC on 25<sup>th</sup> July.

### Acknowledgements

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### 5. Conclusions

- ❑ Low values of coherences have been found for all temporal scales in nocturnal conditions. Only vertical velocity at 1.5m and temperature at 10 m have got some correlation in individual scales
- ❑ A good behavior in the coherence spectrum has been observed between temperature of the two levels in near transition conditions, maybe due to the observed trend.
- ❑ MRFD analysis shows mainly large scales in the NBL, probably driven by gravity waves, while before sunset developed turbulence is clearly present.

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