ESTIMATING THE GLOBAL TEMPERATURE CHANGE BY MEANS OF FUZZY LOGIC MODELS OBTAINED FROM A SIMPLE CLIMATE MODEL

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ABSTRACT
In this work a simple box model of the ocean-atmosphere system is used to asses the response of the simulated global mean temperature to expected changes in the surface thermal forcing at the year 2000, as well as to variations of two key parameters, namely ocean thermal diffusivity and the atmospheric feedback. Such experiments provide the input data needed to build fuzzy logic models that are able to deal with the uncertainties associated to the model parameters. Two fuzzy logic approaches are presented in this article. The first approach is the core of the fuzzy logic, i.e. the Extension Principle (EP), which was postulated by Zadeh as the basis to extend the foundations of classical set theory to fuzzy set theory. The second approach is the Adaptive Network based Fuzzy Inference System (ANFIS), which is a hybrid neuro-fuzzy system for function approximation. Both fuzzy models predict an increase of temperature at year 2100 comparable to that predicted by the box model. These fuzzy models are able to explain the behaviour of the ocean-atmosphere system by means of human understandable rules that contain the uncertainty associated with the problem. The use of fuzzy logic for modelling global temperature change is an interesting approach since it treats uncertainty, which is inherent in science, as part of the model, not trying to avoid it, as classical approaches do.

Key words: Global Warming, Climate Modelling, Fuzzy Logic, Extension Principle, ANFIS.

RESUMEN
En este trabajo se utiliza un modelo de cajas del sistema océano-atmósfera para estudiar la respuesta de la temperatura promedio global a cambios en el forzamiento radiativo y a la variación de dos parámetros importantes del modelo: la difusividad térmica del océano y la sensitividad de la atmósfera (procesos de retroalimentación). A partir de los campos de temperatura obtenidos, se construyen dos modelos basados en lógica difusa. La primera aproximación es el corazón de la lógica difusa, es decir el Principio de Extensión (PE) postulado por Zadeh como la base para extender los fundamentos de la teoría de conjuntos clásica a la teoría de conjuntos difusos. El sistema de inferencia difuso basado en una red adaptativa (ANFIS) es la segunda aproximación de lógica difusa escogida en esta investigación. Con ambas aproximaciones se obtienen buenos resultados. El uso de la lógica difusa para modelar el cambio global de la temperatura es una aproximación interesante pues trata la incertidumbre, que es inherente en la ciencia, como parte del modelo y no intenta evitarla, como hacen las aproximaciones clásicas.

Palabras clave: Calentamiento Global, Modelado del Clima, Lógica difusa, Principio de Extensión, ANFIS.

1. INTRODUCTION

The global climate is a highly complex system in which take place many physical, chemical, and biological processes, in a wide range of space and time scales. These processes are simulated by global circulation models, which are computer models based on the fundamental laws of physics and they are the principal tool for predicting the response of the climate to increases in greenhouse gases. With the increase of computational resources, complex global models are frequently being used to assess the response of the climate system to the projected increase in the amount of greenhouse gases. All model experiments point to global warming through the coming centuries. These models, however, are not perfect representations of reality because, among other reasons, they do not include important physical processes (e.g. ocean eddies, gravity waves, atmospheric convection, clouds and small-scale turbulence) that are known to be key aspects of the climate system but that are too small or fast to be explicitly modelled (WILLIAMS, 2005). In addition, the high complexity of the climate system represents, by itself, a crucial constraint in the prediction of future climate change. Therefore, even the most complex climate models are unable to project how climate will change with certainty, as it is reflected in the wide range of temperature increase reported by the IPCC 4AR (IPCC, 2007).

A sophisticated computer models needs a lot of computing resources. Consequently, in order to produce climate projections for centuries into the future, it is required either a very powerful computer or a less complex model. Simple models of the climate system have been developed and used to gain physical insight into major features of the behaviour of the climate system. These simple models have also been frequently used to conduct sensitivity studies and to produce climate projections for a range of assumptions about emissions of carbon dioxide and other greenhouse gases. By using this kind of models we gain an understanding of the importance of a certain parameter (for example, thermal diffusivity of the ocean) by changing that parameter in the simple climate model and observing the changes in the model predictions (for example, changes in surface temperature).

Fuzzy set theory and fuzzy logic are very powerful tools for managing uncertainties inherent to complex systems. Fuzzy systems have demonstrated their ability to solve different kind of problems like control (e.g. WATANABE et al., 2005) and have been successfully applied to a wide range of applications, i.e. signal and image processing (BLOCH, 2005) and medical applications (NEBOT et al., 2003), etc. To the authors' knowledge, there are very few studies that apply fuzzy logic approaches to study the global temperature change problem. SCHERM (2000) use a fuzzy representation of the temperature and precipitation changes in a climate-pest model. This research does not study the global temperature change in itself, but its influence on plant pests and demonstrates, in a feasible way, the importance of dealing with uncertainties in order to achieve reliable impact assessment and that fuzzy arithmetic allows to explicitly include uncertainty in models of diverse complexity.

In this research, we use a simple box model of the ocean-atmosphere to asses the response of the global mean temperature to changes in the thermal forcing and two model parameters. This model depends on a small number of parameters which are treated directly as fuzzy logic sets. Then, by means of the Zadeh's EP, we obtain as a result the temperature increase ranges. In other words, the introduced method allow us to see how the uncertainties associated with the model parameters are propagated throw the fuzzy model and give as a result the uncertainty ranges associated with the temperature increase. Finally, a hybrid neuro-fuzzy model, which is able to optimize automatically the fuzzy parameters, is also used for global temperature change predictions. Neither of the models proposed in this study are based on probabilities but are rather fuzzy models that represent uncertainties as membership functions.

2. GLOBAL TEMPERATURE CHANGE EXPERIMENT

In this section, we use a box model of the ocean-atmosphere to determine whether this simple model is able to reproduce the wide range of temperature increase reported by the IPCC, when plausible model parameters and surface forcing are used.

2.1. The ocean-atmosphere model

The ocean-atmosphere system is represented in this work by using a simple energy balance model consisting of four boxes: two boxes are used to represent the atmosphere, one of them is over land with zero heat capacity, and the other one is over the ocean; the last two boxes represent the oceanic mixed layer coupled to a diffusive ocean (Fig. 1, left panel).

The analytical solution of this kind of model can be found in WIGLEY and SCHLESINGER (1985). The brief description given here follows closely that of McGUFFIE and HENDERSON-SELLERS (2005). The heating rate of the mixed layer is calculated by assuming a constant depth in which the temperature difference (ΔT), associated with some perturbation, changes in response to: changes in the surface thermal forcing (ΔQ); the atmospheric feedback, which is expressed in terms of a climate feedback parameter (λ); leakage of energy from the mixed layer to the deeper ocean (ΔM). This energy flux is used as an upper boundary condition for the diffusive deep ocean in which the thermal diffusion coefficient (K) is assumed to be a constant.

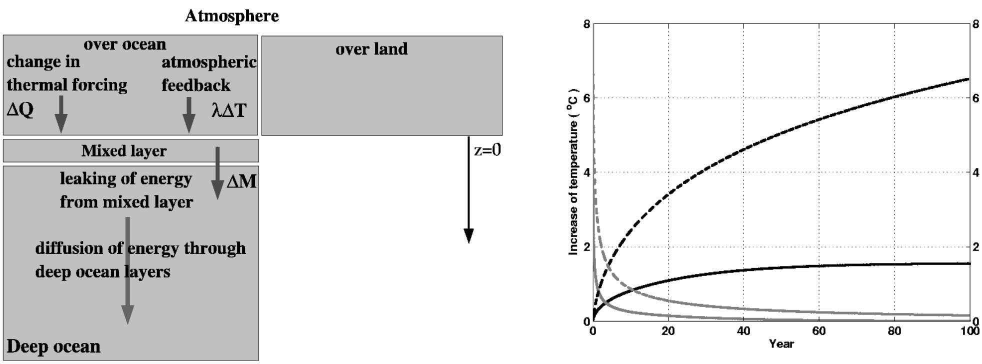

Figure 1: Box-diffusion model of the atmosphere-ocean system (left panel). Two results of the numerical integration of equations (1) and (2) (right panel).

The equations describing the rates of heating in the two layers are:
For the mixed layer, with total heat capacity C_m,

$$C_m \frac{d\Delta T}{dt} = \Delta Q - \lambda \Delta T - \Delta M \tag{1}$$

For the deeper ocean layer,

$$\frac{\partial \Delta T_0}{\partial t} = K \frac{\partial^2 \Delta T_0}{\partial^2 z} \tag{2}$$

At the interface between the surface and the deeper layers, there is an energy source which acts as a surface boundary condition (2). Following to McGUFFIE and HENDERSON-SELLERS (2005), a simple parameterization is used by imposing continuity between the mixed-layer temperature change (ΔT) and the deeper-layer temperature change evaluated at the interface, $\Delta T_0(z=0,t)$, i.e. $\Delta T_0(0,t) = \Delta T(t)$. With this formulation, ΔM can be calculated from

$$\Delta M = -\gamma \rho_w c_w K \left\{ \frac{\partial \Delta T_0}{\partial z} \right\} \Bigg|_{z=0} \tag{3}$$

and used in (1). In the last equation, γ is the parameter utilized to average over land and ocean (values between 0.72 and 0.75), ρ_w is the water density and c_w is its specific heat capacity.

2.2. The numerical experiments

Equations (1) and (2) are integrated numerically for a period of 100 years using a forward Euler scheme and a vertical grid for the deep ocean. All model experiments are performed using a time step of one day and a vertical grid with 100 points and a spacing of 5 m, which represents a deep ocean layer of 500 m. The internal model parameters and the change in thermal forcing vary as follows: λ varies from 0 to 4 $Wm^{-2}K^{-1}$, with increments of 0.25; K varies from 10^{-4} to $10^{-5} m^2 s^{-1}$, with increments of 0.5×10^{-5}; ΔQ varies from 0 to 8 Wm^{-2}, with increments of 0.5. A total of 6069 integrations (each one corresponding to a combination of the varying internal model parameters and the thermal forcing) are carried out over the 100-year period. After 100 years of integration, the model is not at thermal equilibrium when a low value of K is used. In the case of a high K value, the equilibrium is reached after 60 years of integration. The combination of low values of K and λ results in the large temperature increase for a given ΔQ. This range of temperatures increase agrees with that reported by the IPCC (Fig. 1, right panel).

3. FUZZY LOGIC BASIC CONCEPTS

As Klir stated in his book (KLIR and ELIAS, 2002), the view of the concept of *uncertainty* has been changed in science over the years. The traditional view looks to uncertainty as undesirable in science and should be avoided by all possible means. The modern view is tolerant of uncertainty and considers that science should deal with it because it is part of the real world. This is especially relevant when the goal is to construct models. In this case, allowing more uncertainty tends to reduce complexity and increase credibility of the resulting model. The recognition by the researchers of the important role of uncertainty mainly occurs with the first publication of the fuzzy set theory, where the concept of objects that have not precise boundaries (fuzzy sets) is introduced (ZADEH, 1965).

Fuzzy logic, based on fuzzy sets, is a superset of conventional two-valued logic that has been extended to handle the concept of partial truth, i.e. truth values between completely true and completely false. In classical set theory, when **A** is a set and **x** is an object, the proposition "**x** is a member of **A**" is necessarily true or false, as stated on equation 4,

$$A(x) = \begin{cases} 1 & for\ x \in A \\ 0 & for\ x \notin A \end{cases} \tag{4}$$

whereas, in fuzzy set theory, the same proposition is not necessarily either true or false, it may be true only to some degree. In this case, the restriction of classical set theory is relaxed allowing different degrees of membership for the above proposition, represented by real numbers in the closed interval [0,1], i.e. $A: X \rightarrow [0,1]$. Fig. 2 presents this concept graphically

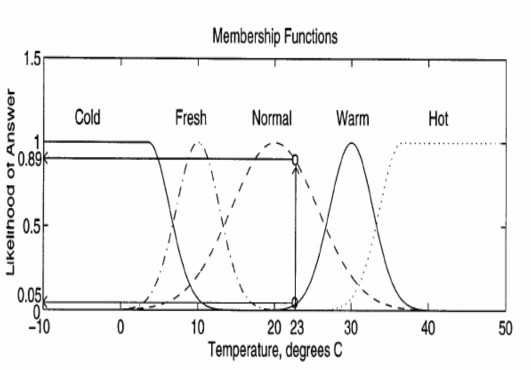

Figure 2: Gaussian membership functions of a quantitative variable representing ambient temperature

Fig. 2 illustrates the membership functions of the classes: *cold*, *fresh*, *normal*, *warm*, and *hot,* of the ambient temperature variable. A temperature of 23°C is a member of the class *normal* with a grade of 0.89 and a member of the class *warm* with a grade of 0.05. The definition of the membership functions may change with regard to who define them. For example, the class normal for ambient temperature variable in Mexico City can be defined as it is shown in Fig. 2. The same class in Anchorage, however, will be defined more likely in the range from -8°C to -2°C. It is important to understand that the membership functions are not probability functions but subjective measures. The opportunity that brings fuzzy logic to represent sets as degrees of membership has a broad utility. On the one hand, it provides a meaningful and powerful representation of measurement uncertainties, and, on the other hand, it is able to represent efficiently the vague concepts of natural language.

At this point, the question is, once we have the variables of the system that we want to study described in terms of fuzzy sets, what can we do with them? The membership functions are the basis of the fuzzy inference concept. The compositional rule of inference is the tool used in fuzzy logic to perform approximate reasoning. Approximate reasoning is a process by which an imprecise conclusion is deduced from a collection of imprecise premises using fuzzy sets theory as the main tool.

The compositional rule of inference translates the *modus ponens* of the classical logic to fuzzy logic. The generalized *modus ponens* is expressed by:

$$\begin{array}{l} \text{Rule: If } X \text{ is } A \text{ then } Y \text{ is } B \\ \text{Fact: } X \text{ is } A' \\ \hline \text{Conclusion: } Y \text{ is } B' \end{array}$$

where, *X* and *Y* are variables that take values from the sets X and Y, respectively, and *A*, *A*' and *B*, *B*' are fuzzy sets on X and Y, respectively. Notice that the *Rule* expresses a fuzzy relation, *R*, on X x Y.

Then, if the fuzzy relation, *R*, and the fuzzy set *A*' are given, it is possible to obtain *B*' by the compositional rule of inference, given by equation 5,

$$B'(y) = \sup_{x \in X} \min \left[A'(x), R(x, y) \right] \tag{5}$$

where **sup** stands for supremum (least upper bound) and **min** stands for minimum. When sets X and Y are finite, **sup** is replaced by the maximum operator, **max**. Fig. 3 illustrates in a simplified way the compositional rule of inference graphically.

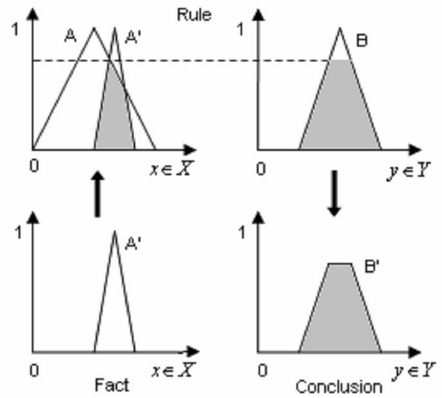

Figure 3: Simplified graphical representation of the compositional rule of inference

It is not the purpose of this section to bring a deep introduction to the fuzzy logic approach but to transmit the key concepts to understand the fuzzy models presented later. For more information see KLIR and ELIAS (2002).

4 EXTENSION PRINCIPLE MODEL

Zadeh says that rather than regarding fuzzy theory as a single theory, we should regard the conversion process from binary to membership functions as a methodology to generalize any specific theory from a crisp (discrete) to a continuous (fuzzy) form. The extension principle enables us to extend the domain of a function on fuzzy sets, i.e. it allows us to determine the fuzziness in the output given that the input variables are already fuzzy. Therefore, it is a particular case of the compositional rule of inference (see section 3).

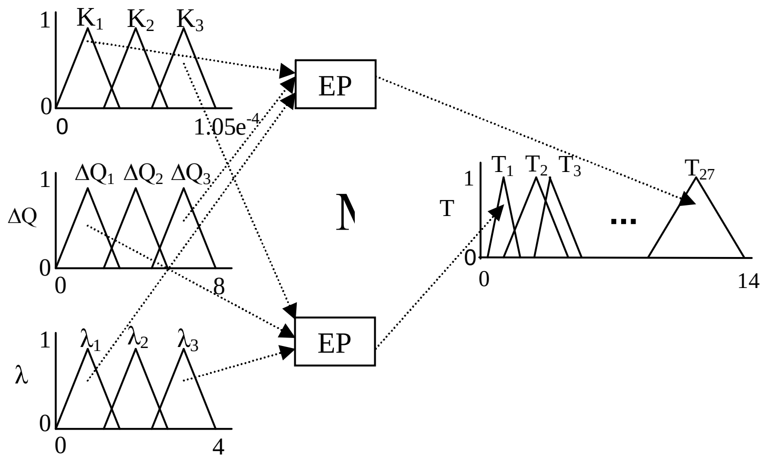

Figure 4: Extension principle model for the global temperature change problem

In this research we propose to construct a fuzzy model of the global temperature change on the basis of the extension principle. To this end, we start from the input variables: ocean diffusivity, K, change in the surface thermal forcing, ΔQ, and atmospheric feedback, λ, and the temperature change function, already described in section 2. The inherent uncertainty associated to the input variables supports the idea of dealing with fuzzy variables instead of crisp variables. Therefore, K, ΔQ and λ variables were converted into fuzzy variables by defining three fuzzy sets, i.e. *low*, *medium* and *high* for each one of them, as shown in the left hand side of Fig. 4. These fuzzy sets were defined in a "manual" way, i.e. using expert knowledge and experience.

The extension principle is then applied to transform each fuzzy triple (K_i, ΔQ_j, λ_r), in a fuzzy set of the temperature change variable. Notice that we have 27 triples of fuzzy input sets and, therefore, 27 fuzzy sets are obtained representing the conclusion, i.e. temperature change variable, as shown in the right hand side of Fig. 4. The extension principle when three input variables are available is described by equation (6). T_s is the temperature fuzzy set extended from the three input fuzzy sets K_i, ΔQ_j and λ_r. In the application at hand, as illustrated in Fig. 4, the extension principle is applied 27 times, to obtain each of the temperature fuzzy sets associated to each fuzzy input tuple.

$$T_s = \max_{T_s = f(k_i, \Delta Q_j, \lambda_r)} \min \left[K_i, \Delta Q_j, \lambda_r \right] \tag{6}$$

For instance, the output fuzzy set T_{27}, is obtained when using the extension principle of equation 6 with the input fuzzy sets K_1, ΔQ_3 and λ_1. Notice that we need to use **max** because equal temperature values can be found in different tuples. The fuzzy rules are described by equation 7:

$$\begin{array}{l} \text{Rule}_1: \quad \textbf{If } K \text{ is } K_1 \textbf{ and } \Delta Q \text{ is } \Delta Q_3 \textbf{ and } \lambda \text{ is } \lambda_1 \textbf{ then } T \text{ is } T_{27} \\ \qquad\qquad\qquad\qquad \vdots \\ \text{Rule}_{27}: \textbf{If } K \text{ is } K_3 \textbf{ and } \Delta Q \text{ is } \Delta Q_1 \textbf{ and } \lambda \text{ is } \lambda_3 \textbf{ then } T \text{ is } T_1 \end{array} \tag{7}$$

The model presented in this section is validated by predicting the temperature change of 357 data points. In this case, the Mamdani scheme (NAUCK *et al.*, 1997) is chosen to perform the inference process (see section 2). Figure 5 shows the “real” temperature change values, i.e. the data obtained from equations 1 and 2 (thick-gray line), and the predicted values obtained by means of the extension principle (black line) when the ΔQ variable is set to 13. As can be seen from this figure, the predictions obtained by the extension principle model follow fairly well the “real” data.

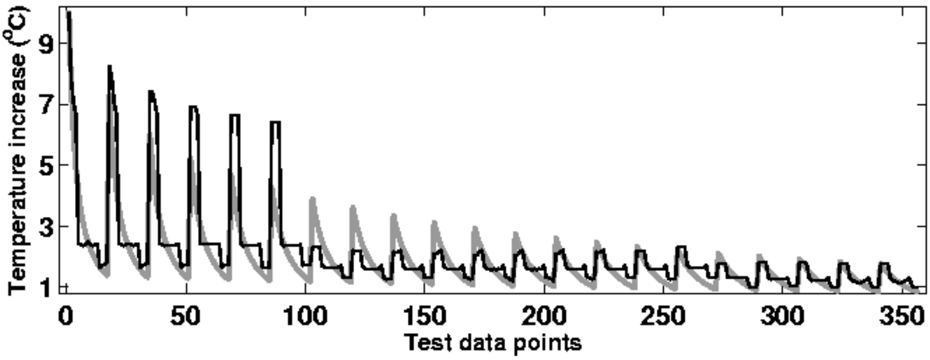

Figure 5: Temperature change prediction results of the Extension Principle model for a fixed value of the change in the surface thermal forcing (ΔQ =13)

The root mean square error of the predictions obtained for the test data set when using the extension principle model is 0.73. The implementation of the proposed model has been done under the Matlab environment.

5. ANFIS model

Fuzzy logic, together with other computational approaches such as neural networks, evolutionary computation, machine learning, and probabilistic reasoning compose the concept defined by Zadeh as Soft Computing. Soft computing differs from conventional (hard) computing in that, unlike hard computing, it is tolerant of imprecision, uncertainty, partial truth, and approximation. The role model for soft computing is the human mind. The guiding principle of soft computing is: Exploit the tolerance for imprecision, uncertainty, partial truth, and approximation to achieve tractability, robustness, and low solution cost (ZADEH, 1994).

In this section a soft computing method, based on neural networks and fuzzy logic, called ANFIS, is used to develop a model of the global temperature change. The Adaptive Network based Fuzzy Inference System (ANFIS), developed by Jang, is one of the most popular hybrid neuro-fuzzy system for function approximation (NAUCK *et al.*, 1997) and its main originality is that performs an automatic optimization of the input variables membership functions. Why is this interesting? In the extension principle model presented in section 4, the membership functions of the input variables, K, ΔQ and λ where chosen in a manual way from the basis of authors' knowledge and expertise. It would be very useful, however, to have an automatic way of finding the best definitions of the membership functions with respect the prediction performance of the constructed model. In this way, it is guaranteed that the more appropriate and useful representation of the input variables is found, i.e. the uncertainties associated to the temperature change parameters are characterized in the best possible way.

ANFIS represents a Sugeno-type neuro-fuzzy system. A neuro-fuzzy system is a fuzzy system that uses learning methods derived from neural networks to find its own parameters, as the membership functions of the input variables. A relevant thing, here, is that the learning process is not knowledge-based (as was the case in the extension principle model), but data-driven. ANFIS is a function of the Fuzzy toolbox of Matlab.

In order to understand the ANFIS model developed in this research for the global temperature change variable, it is necessary to look closer to the modus operandi of a Sugeno fuzzy model. The main characteristic of the Sugeno inference system is that the consequent or output of the fuzzy rules is not a fuzzy variable but a function, as shown in equation 8.

$$\begin{aligned} &\text{Rule}_1\text{:} \quad \text{If A is } A_1 \text{ and B is } B_1 \text{ then } z = p_1 * a + q_1 * b + r_1 \\ &\text{Rule}_2\text{:} \quad \text{If A is } A_2 \text{ and B is } B_2 \text{ then } z = p_2 * a + q_2 * b + r_2 \end{aligned} \tag{8}$$

In the application at hand the ANFIS model is composed of 27 Sugeno rules, as the ones described in equation 9, due to the fact that 3 membership functions were used to represent each input variable. The ANFIS parameters are optimized by using a set of 5395 data points for which less than 2 minutes of a HP Compaq computer time is needed. As in the extension principle model, the ANFIS model is validated by predicting the temperature change of the 357 data used in section 4. These data and the prediction obtained by means of the ANFIS model are shown in Fig. 6. As can be seen from the plot, ANFIS is able to predict very accurately the temperature change test values, with a very low root mean square error of 0.23. A comparison of figures 5 and 6 indicates that the ANFIS model is able to predict more accurately the real signal than the EP model. Notice that in Fig. 5 the ΔQ variable is fixed to a value of 13, whereas in Fig. 6 all the possible values of the three input variables are used in the ANFIS model to predict the temperature change. Both models represent accurately the expected global warming at year 2100 and are able to present the behavior of the system by means of understandable rules that contain the uncertainty associated to the problem.

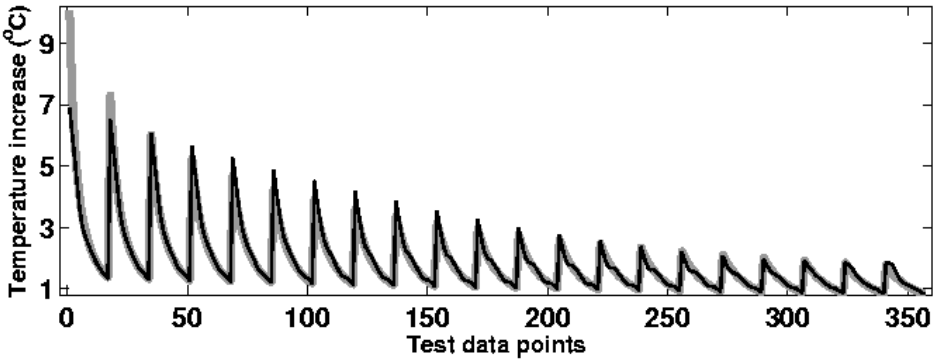

Figure 6: Temperature change prediction results of the ANFIS model

5. CONCLUSIONS

In this paper a simple box model of the ocean-atmosphere is used to asses the response of the coupled system to variations of two important internal parameters of the model, namely the thermal diffusion coefficient and the atmospheric feedback expressed in terms of a climate feedback parameter. When a combination of plausible values of these model parameters, in addition to the expected thermal forcing associated with the global warming, the box model reproduces satisfactorily the wide range of temperature increase reported by the IPCC.

From the temperature increase calculated with the box model, two fuzzy logic models based on the extension principle (EP) and the adaptive network based fuzzy inference system (ANFIS) are built. These fuzzy models are able to deal with the uncertainties associated with the parameters of the deterministic model and the thermal forcing. The EP and ANFIS models are able to predict accurately the global temperature increase in the year 2100. The differences between the box model and ANFIS are lower than those observed between the box model and EP due to the fact that ANFIS makes an automatic optimization of the fuzzy parameters. Both fuzzy models are able to explain the behaviour of the coupled ocean-atmosphere system by means of understandable rules that contain the uncertainty associated to the problem.

The fuzzy models presented in this paper are simpler than the box model and are much more understandable from a policy maker point of view.

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