

FUZZY RULES MODEL FOR GLOBAL WARMING DECISION SUPORT

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ABSTRACT

In this work a simple box model of the ocean-atmosphere is used to asses the response of the coupled system to the projected increase in the amount of carbon dioxide, by varying internal model parameters, within plausible ranges, as well as the thermal forcing associated with the greenhouse gases. The values of temperature increase are used to build fuzzy logic models based on the Fuzzy Inductive Reasoning (FIR) methodology that are able to deal with the uncertainties associated to the box model parameters. FIR is a data driven methodology that uses fuzzy and pattern recognition techniques to infer system models and to predict their future behavior. The fuzzy models developed are able to predict accurately the global temperature change in year 2100. Moreover, in this research an extension of the FIR methodology, named LR-FIR (Linguistic rules in FIR), is presented and used to obtain fuzzy rules models which synthesize in an understandable (human thinking) way the global temperature change dynamics. This characteristic and the simplicity of the proposed models make them to become very useful tools for decision making related to global warming.

Key words: Global Warming, Decision Making, Fuzzy Inductive Reasoning, Fuzzy Logic.

RESUMEN

En este trabajo se utiliza un modelo simple del sistema océano-atmósfera para obtener valores de temperatura promediados globalmente. En este modelo, la temperatura es una función del calor agregado al sistema, de la sensibilidad de la atmósfera y de la difusividad del océano. A partir de los campos de temperatura obtenidos, se construye un modelo basado en lógica difusa, concretamente usando la metodología del Razonamiento Inductivo Difuso (FIR, por las siglas en inglés), el cual es capaz de predecir el cambio de temperatura global con una gran precisión. Sin embargo, el modelo FIR no permite una interpretación suficientemente sencilla de la dinámica del sistema para que sea útil a los tomadores de decisiones. A partir del modelo FIR es posible extraer un modelo reducido basado en un conjunto de reglas difusas el cual además de ser más simple que el modelo determinista incluye las incertidumbres de los parámetros utilizados en ese modelo. Esta característica permite, entre otras cosas, su aplicación a la toma de decisiones relacionadas con el calentamiento global. Las reglas difusas que se obtienen a partir del modelo cuantitativo del sistema océano-atmósfera presentan la dinámica fundamental del sistema en lenguaje humano, de manera que resulta una herramienta de análisis de posibles situaciones futuras. En esta investigación se presentará dicho modelo y se argumentará el uso de éste como soporte a la toma de decisiones.

Palabras clave: Calentamiento Global, Toma de Decisiones, Razonamiento Inductivo Difuso, Lógica Difusa.

1. INTRODUCTION

The projections of future global temperature changes ranged from 1.4°C to 5.8°C in the case of the IPCC's TAR report (IPCC, 2001). More recently, these projections indicate that temperatures would cover a range spanning from 1.2°C to 4°C, but that temperatures of more than 6°C could not be ruled out. From the point of view of a policy maker the results of the 3rd and 4th IPCC's assessments regarding the projection of global or for that matter regional temperature increases pose a problematic analysis due to the large difference in values, i.e. up to a 600 percent. These differences are the reflection of the uncertainties that have to be dealt with by the scientists and ultimately the policy makers. The sources of the uncertainty have diverse origins. Complex general circulation models are the principal tool for predicting the response of the climate to increases in greenhouse gases. The most sophisticated of these models include the interaction of the atmosphere with the oceans and with the surface of the Earth, including plants and other ground cover. When a complex general circulation model is used, a big number of parameters exist (clouds, sensible heat, latent heat, etc.), that can be different from one model to another and produce different temperature values. In addition, a multi-decadal climate model prediction implies a lack of constraint from real-world observed values of the model state variables. Therefore, a model produces results within an intrinsic range of uncertainty, which increases when results of different models, driven by the same boundary conditions but using different numerical architectures, are compared. For instance, the HADLEY (POPE *et al.*, 2000) and the ECHAM (ROECKNER *et al.*, 1996) models.

A sophisticated computer models needs a lot of computing resources. Consequently, in order to produce climate projections for centuries into the future, it is required either a very powerful computer or a less complex model. Simple models of the climate system have been developed and used to gain physical insight into major features of the behaviour of the climate system. These simple models have also been frequently used to conduct sensitivity studies and to produce climate projections for a range of assumptions about emissions of carbon dioxide and other greenhouse gases. There is a large reason for simple models to remain important, not only for science but for policy-making. Simple models offered a variety of other approaches, more comprehensible and easier to verify within their special domains. This characteristic makes them a valuable tool to add flexibility and plausibility to decisions about future policies.

Policy makers argue that they need more information than what is provided by climate change science. They do not know how to handle the uncertainty because many of them confuse uncertainty with ignorance. Therefore, an interesting and useful option for policy makers is to have available models that have already the uncertainty incorporated as part of the model and that the information is presented to them in human natural language. Fuzzy logic is the only approach that allows to reach this goal. Fuzzy set theory and fuzzy logic are very powerful tools for managing uncertainties inherent to complex systems. Fuzzy systems have demonstrated their ability to solve different kind of problems like control (WATANABE *et al.*, 2005), modeling (ESCOBET *et al.*, 2007) or classification (KUNCHEVA, 2000), and have been successfully applied to a wide range of applications, i.e. signal and image processing (BLOCH, 2005), industrial applications (DOTE and OVASKA, 2001), medical applications (NEBOT *et*

al., 2003) etc. There are very few works, however, that apply fuzzy logic approaches to study the global temperature change problem. From them the works of (ROMANO *et al.*, 2004, CASTAÑEDA *et al.*, 2006) and (HALL *et al.*, 2007) are, maybe, the more relevant.

The idea of this research is to use a robust, simple climate model to reproduce not only a unique temperature but directly the range of temperatures that appear in the TAR (IPCC, 2001) as well as in the four evaluation reports of the IPCC 4AR (IPCC, 2007). The proposed model depends on a small number of parameters with well known ranges of variations and that are treated directly as fuzzy logic sets. The main goal of this research is to use the fuzzy inductive reasoning (FIR) methodology and an extension of it, named LR-FIR (Linguistic rules in FIR), to obtain fuzzy rules models which synthesize in an understandable (human thinking) way the global temperature change dynamics. This characteristic and the simplicity of the proposed models make them very useful tools for decision making related to global warming.

2. GLOBAL TEMPERATURE CHANGE EXPERIMENT

2.1. The ocean-atmosphere model

The ocean-atmosphere system is represented in this work by using a simple energy balance model consisting of four boxes: two boxes are used to represent the atmosphere, one of them is over land with zero heat capacity, and the other one is over the ocean; the last two boxes represent the oceanic mixed layer coupled to a diffusive ocean (see Fig. 1).

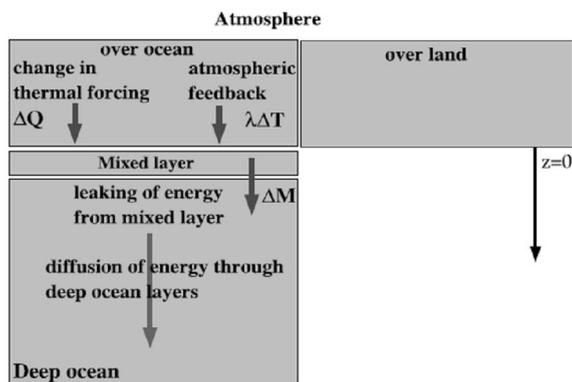


Figure 1: Ocean-atmosphere system using a simple energy balance model

The analytical solution of this kind of model can be found in WIGLEY and SCHLESINGER (1985). The brief description given here follows closely that of McGUFFIE and HENDERSON-SELLERS (2005). The heating rate of the mixed layer is calculated by assuming a constant depth in which the temperature difference (ΔT), associated with some perturbation, changes in response to: changes in the surface thermal forcing (ΔQ); the atmospheric feedback, which is expressed in terms of a climate feedback parameter (λ); leakage of energy from the mixed layer to the deeper ocean (ΔM). This energy flux is used as

an upper boundary condition for the diffusive deep ocean in which the thermal diffusion coefficient (K) is assumed to be a constant.

The equations describing the rates of heating in the two layers are:

1) for the mixed layer, with total heat capacity C_m ,

$$C_m \frac{d\Delta T}{dt} = \Delta Q - \lambda \Delta T - \Delta M \quad (1)$$

2) for the deeper ocean layer,

$$\frac{\partial \Delta T_0}{\partial t} = K \frac{\partial^2 \Delta T_0}{\partial z^2} \quad (2)$$

Eq. 2 is resolved numerically using a vertical grid. At the interface between the surface and the deeper layers, there is a energy source which acts as a surface boundary condition for this partial differential equation. Following to McGUFFIE and HENDERSON-SELLERS (2005), a simple parameterization is used by imposing continuity between the mixed-layer temperature change (ΔT) and the deeper-layer temperature change evaluated at the interface, $\Delta T_0(z = 0, t)$, i.e.

$$\Delta T_0(0, t) = \Delta T(t) \quad (3)$$

With this formulation, ΔM can be calculated from

$$\Delta M = -\gamma \rho_w c_w K \left. \left\{ \frac{\partial \Delta T_0}{\partial z} \right\} \right|_{z=0} \quad (4)$$

and used in (1). In the last equation, γ is the parameter utilized to average over land and ocean (values between 0.72 and 0.75), ρ_w is the water density and c_w is its specific heat capacity.

2.2. The numerical experiments

The equations are integrated numerically for a period of 100 years using a forward Euler scheme and a vertical grid for the deep ocean. All model experiments are performed using a time step of one day and a vertical grid with 100 points and a spacing of 5 m, which represents a deep ocean layer of 500 m. The internal model parameters and the change in thermal forcing vary as follows: λ varies from 0 to $4 \text{ Wm}^{-2}\text{K}^{-1}$, with increments of 0.25; K varies from 10^{-4} to $10^{-5} \text{ m}^2\text{s}^{-1}$, with increments of 0.5×10^{-5} ; ΔQ varies from 0 to 8 Wm^{-2} , with increments of 0.5. A total of 6069 integrations (each one corresponding to a combination of the varying internal model parameters and the thermal forcing) are carried out over the 100-year period. After 100 years of integration, the model is not at thermal equilibrium when a low value of K is used. In the case of high thermal diffusivities, the equilibrium is reached after 60 years of integration. The combination of low values of K and λ results in the large temperature increase for a given ΔQ . This range of temperature increase agrees with that reported by the IPCC.

3. FUZZY LOGIC APPROACH

Fuzzy logic is a superset of conventional boolean logic that has been extended to handle the concept of partial truth, i.e. truth values between completely true and completely false. It was introduced by Zadeh in the 1960's as a means to model the uncertainty of natural language. We aim to use the fuzzy logic approach to the problem of global temperature change modelling because it allows to treat the uncertainty inherent in this system as part of the model, not trying to avoid it, as classical approaches do. In this paper it is intended to perform approximate reasoning by means of a powerful fuzzy logic methodology named Fuzzy Inductive Reasoning (FIR). Approximate reasoning is a process by which an imprecise conclusion is deduced from a collection of imprecise premises using fuzzy sets theory as the main tool.

3.1. Fuzzy Inductive Reasoning (FIR)

The Fuzzy Inductive Reasoning methodology is a mathematical tool for modeling and simulation of complex systems. FIR is based on systems behavior rather than on structural knowledge. It is able to perform a selection of the system relevant variables and to obtain the causal and temporal relations between them in order to infer the future behavior of that system. It also has the ability to describe systems that cannot easily be described by classical mathematics (e.g. differential equations), i.e. systems for which the underlying physical laws are not well understood. It offers a model-based approach to predicting either univariate or multi-variate time series. A FIR model is a qualitative, non-parametric, shallow model based on fuzzy logic. FIR is executed under the Visual-FIR platform that runs under the Matlab environment (ESCOBET *et al.*, 2007). In this section a brief explanation of the main aspects of FIR methodology are presented in order to facilitate the user the understanding of the FIR models presented in the next section. The FIR methodology performs two main tasks, i.e. the *model identification* and the *prediction* during which the model previously identified is used to estimate the future behavior of the system.

In order to identify the best model from the data available, it is first necessary to *discretize* the system data. To this end, the quantitative values are converted into qualitative data. For example, an ambient temperature of 23°C is converted to the class *normal* with a grade of membership to this class of 0.895, as expressed in figure 2. The class value represents a discretization of the original real-valued variable. The fuzzy membership value denotes the level of confidence expressed in the class value chosen to represent a particular quantitative value. The *model identification* function is responsible for finding causal spatial and temporal relations between variables that offer the best likelihood for being able to predict the future system behavior from its own past, thereby obtaining the best model. The FIR model is composed by its structure or set of relevant variables (called mask) and a set of input/output rules that represent the systems' history behavior (called pattern rule base). A mask denotes a dynamic relationship among qualitative variables. The optimality of the mask is evaluated with respect to the maximization of its forecasting power that is quantified by means of a quality measure, based mainly on the Shannon entropy. Once the best mask has been identified, it can be applied to the qualitative data matrices that were previously obtained in the discretization process, resulting in a pattern rule base. The full FIR model is composed of the optimal mask and the pattern rule base.

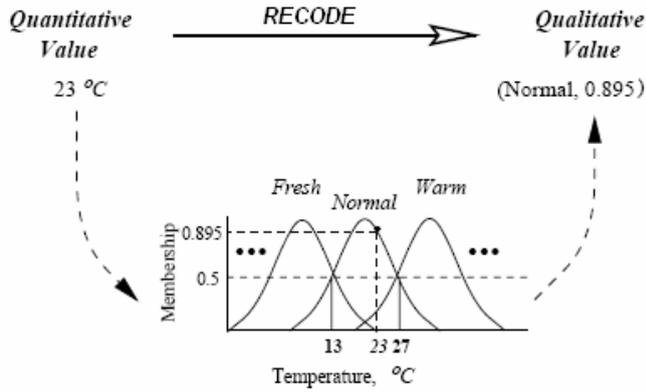


Figure 2: Recode function for ambient temperature variable

Once the FIR model is available, a prediction of future output states of the system can take place using the FIR inference engine. The FIR inference engine is based on a variant of the k-nearest neighbor rule, i.e., the 5-NN pattern matching algorithm. The forecast of the output variable is obtained by means of the composition of the potential conclusion that results from firing the five rules, whose antecedents best match the actual state. The contribution of each neighbor to the estimation of the prediction of the new output state is a function of its proximity. A detailed description of FIR methodology and Visual-FIR platform can be found in (NEBOT *et al.*, 2003; ESCOBET *et al.*, 2007).

3.2. FIR models for the global temperature change

In this research the input variables of the system are the change in the surface thermal forcing, ΔQ , the climate feedback parameter, λ , and the turbulent ocean diffusion coefficient, K , whereas the output variable is the global temperature in year 2100. The FIR model is trained starting from the training data set that consist of 5395 data values, for each input and output variables. The data was generated as described in section 2.

As explained before, in order to obtain a FIR model it is first necessary to convert the quantitative data into qualitative data by means of the *discretization* function. In this case, all the 3 input variables are discretized into 3 classes, i.e. low, medium and high, whereas the output variable, is discretized into 5 classes, i.e. very low, low, medium, high and very high. Table 1 shows for each variable the range values that represents each class. These ranges have been defined by experts in climate change. Now that the quantitative variables are converted into fuzzy variables, the FIR models are identified by means of the *optimal mask* function. In this process the structure of the model (mask) is determined and it is used to obtain the pattern rule base. Figure 3 shows the optimal FIR model obtained.

The optimal mask obtained is composed of all the system input variables, i.e. a negative value means that that variable is relevant to predict the output (TEM) values. Therefore, FIR finds that all three input variables are important and that there is not redundancy in them. It is interesting to notice, that the so called pattern rule base is quite large, i.e. 56 rules, avoiding an easy and friendly interpretation of the fuzzy relations involve. From the 56 pattern rules

obtained by FIR, 24 define the class *very low* of the temperature output variable (represented by '1' in figure 3); 17 define the class *low* (represented by '2'); 9 define the class *medium* ('3'); 4 define the class *high* ('4') and 2 define class *very high* (represented by '5' in figure 3).

	<i>Low</i>	<i>Medium</i>	<i>High</i>
<i>K</i>	$[5e^{-6}..3e^{-5}]$	$(3e^{-5}..8e^{-5})$	$(8e^{-5}..1.05e^{-4})$
λ	[0..1]	(1..2]	(2..4]
ΔQ	[0..2.5]	(2.5..5.5]	(5.5..8]

	<i>Very low</i>	<i>Low</i>	<i>Medium</i>	<i>High</i>	<i>Very high</i>
TEM	[0..1.5]	(1.5..3]	(3..5]	(5..7]	(7..13.45]

Table 1: DISCRETIZATION OF SYSTEM VARIABLES: ΔQ , λ AND *K* INTO 3 CLASSES; TEM INTO 5 CLASSES

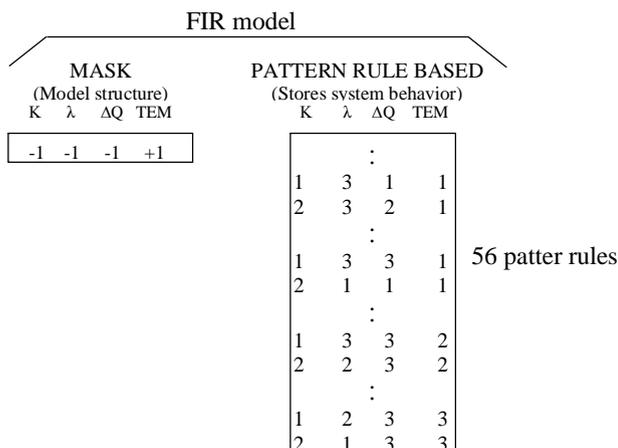


Figure 3: FIR model (mask and an excerpt of the pattern rule base) for the global temperature. Note: in this figure for simplicity reasons the classes low, medium and high are represented by the values '1', '2', and '3' for *K*, λ and ΔQ variables. For TEM variable, low, medium, high and very high are represented by the values '1', '2', '3', '4' and '5', respectively.

The FIR model obtained is very precise when it is used to predict a test data set of 674 values, not used in the training set. As can be seen in figure 4, the real and the predicted values are almost undistinguishable one from each other, being the root mean square error extremely low, i.e. of $5.572e^{-2}$.

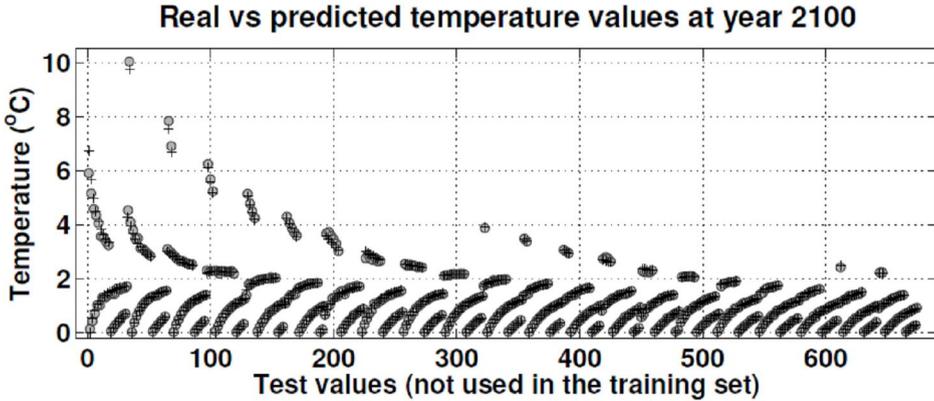


Figure 4: Real ('+') vs. Predicted ('o') test values when using the FIR model of figure 3 to predict the temperature at year 2100

4. RULES FOR UNDERSTANDING

The FIR model presented in the previous section has the ability of holding a high prediction power. The pattern rules interpretability is however quite low, and therefore, it is not a useful tool for decision makers. This paper is mainly focused on the global temperature change modeling for decision support and, therefore, the easy understanding and interpretation by decision makers of the model is of huge importance. In this section a model based on linguistic rules that is directly obtained from the FIR model of section 3 is presented. First, the main characteristics of the algorithm that performs the extraction of linguistic rules from the pattern rules that compose the FIR model are presented. Second, the linguistic rules model obtained for the application at hand is presented and analyzed.

4.1. Linguistic Rules in FIR (LR-FIR)

LR-FIR (LR stands for Linguistic Rules) is an iterative process that compact the pattern rule base identified by FIR. In order to preserve a model that is congruent with those previously identified by FIR, the proposed algorithm is based on its initial discretization, using only the mask features and the pattern rule base obtained (refer to table 1 and figure 3). LR-FIR can be summarized as a set of ordered steps:

1. *Basic compactation.* This is an iterative step that evaluates, one at a time, all the rules in a pattern rule base. The pattern rule base, R , is compacted on the basis of the “knowledge” obtained by FIR. A subset of rules R_c can be compacted in the form of a single rule r_c , when all premises P but one (P_a), as well as the consequence C share the same values. Premises, in this context, represent the input features, whereas consequence is the output feature in a rule. If the subset contains all legal values of P_a , all these rules can be replaced by a single rule, r_c , that has a value of -1 in the premise P_a . A -1 value means that this premise can take any of its possible values.

2. *Improved compactation.* The improved compactation is an extension of the basic one, where a consistent and reasonable minimal ratio, MR , of the legal values should be present in the candidate subset R_c , in order to compact it in the form of a single rule r_c .

The obtained set of rules is subjected to a number of refinement steps: removal of duplicate rules and conflicting rules; unification of similar rules; evaluation of the obtained rules and filtering of rules with low specificity and sensitivity values (this two concepts are introduced later). For a more detailed description of LR-FIR methodology the user is referred to (CASTRO *et al.*, 2007).

4.2. Linguistic rules for the global temperature change

LR-FIR is then used to obtain a set of linguistic rules starting form the FIR model presented in section 3. In this case pattern rules that have equal or less than 4 instances are treated as outliers and the filtering sensitivity value is set to 0.15. Table 2 shows the linguistic rules model. The 56 pattern rules that the FIR model contains are compacted into only 7 linguistic rules.

Logical Rules		
IF λ IS medium-high AND ΔQ IS low-medium THEN TEM IS very low	0.92	0.67
IF λ IS high AND ΔQ IS high THEN TEM IS very low	0.75	0.16
JOINT METRICS <i>Very low temperature</i>	0.75	0.83
IF λ IS medium-high AND ΔQ IS high THEN TEM IS low	0.82	0.56
IF K IS medium-high AND λ IS low AND ΔQ IS high THEN TEM IS low	0.97	0.18
JOINT METRICS <i>Low temperature</i>	0.77	0.89
IF K IS low-medium AND λ IS low-medium AND ΔQ IS medium-high THEN TEM IS medium	0.80	1
JOINT METRICS <i>Medium temperature</i>	0.80	1
IF K IS low AND λ IS low AND ΔQ IS medium-high THEN TEM IS high	0.97	1
JOINT METRICS <i>High temperature</i>	0.97	1
IF K IS low AND λ IS low AND ΔQ IS high THEN TEM IS very high	0.98	1
JOINT METRICS <i>Very high temperature</i>	0.98	1

Table 2: LINGUISTIC RULES MODEL FOR THE GLOBAL TEMPERATURE CHANGE

Notice, that two rules are needed to represent the behaviour of *very low* and *low* temperatures and that only one rule is needed to represent the behaviour of *medium*, *high* and *very high* temperatures. In table 2 the rules have associated two standard metrics, named specificity and sensitivity. Specificity is defined as one minus the ratio of the number of out-of-class data records that the rule identifies to the total number of out-of-class data. Sensitivity is the ratio of the number of in-class data that the rule identifies to the total number of in-class data. Both metrics are in the range [0..1]. It is desirable that both the specificity and the sensitivity achieve large values. As can be seen in table 2 the specificity and sensitivity metrics are very high for

those rules that define *medium*, *high* and *very high* temperatures. A high specificity means that there is a very low grade of false positives, i.e. the rule says that the temperature (TEM) is in a specific class and it is not true. A high sensitivity means that there is a very low grade of false negatives, i.e. the rule says that the temperature (TEM) is not in a specific class and it is not true. Therefore, the last three rules by themselves have high values of both metrics indicating a very good representation of system behaviour for *medium*, *high* and *very high* increments of global temperature at 2100. Also good metrics are obtained for *very low* and *low* temperatures by means of the joint rules.

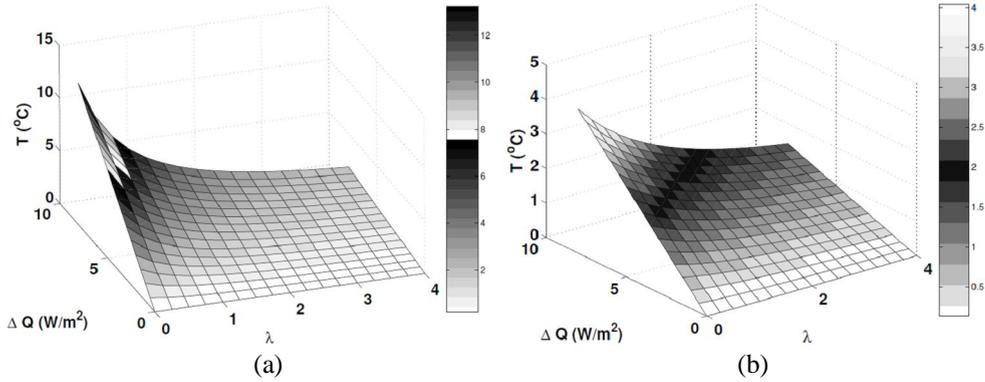


Figure 5: Graphical representation of temperature for year 2100 with respect ΔQ and λ variables: (a) for a fixed K value of *low* and (b) for a fixed K value of *high*.

Figure 5 shows a graphical representation of temperature values for year 2100 with respect variables ΔQ and λ . In the left hand side plot (a) the K variable is fixed to a value of the class *low* whereas in the right hand side plot (b) the K variable is fixed to a value of the class *high*. These figures validate the linguistic rules obtained in this work, presented in table 2. The rules that define the temperature classes *medium*, *high* and *very high* can be evaluated in the surface of figure 5(a). Let us start by the rule that define the *very high* temperature class. When λ is *low* and ΔQ is *high* then the rule tells us that the temperature is very high, i.e. values in between 7°C and 13.45°C. This is directly validated from the surface of figure 5(a). The same can be said from rules that define temperature classes *high* and *medium*. If we select the area of the surface with λ values in the range *low-medium* and ΔQ values in the range *medium-high* we found that the temperature values belong to the class *medium*, i.e. values in between 3°C and 5°C. The rules that define the temperature classes *low* and *very low* can be evaluated in the surface of figure 5(b). The *low* temperatures (values in between 1.5°C and 3°C) are defined by two rules, i.e. when λ is *low* and ΔQ is *high* and when λ is *medium-high* and ΔQ is *high*. The same analysis can be performed for the rules that define temperature class *very low*. The set of 7 logical rules obtained allows the easy understanding of system behavior for decision and policy makers making their work easier.

5. CONCLUSIONS

In this paper a robust, simple climate model is used to reproduce the set of temperature values that are observed in the TAR and 4AR IPCC's reports. From the temperatures obtained from this model a fuzzy logic model based on the Fuzzy Inductive Reasoning (FIR) methodology is built. The FIR model is able to deal with the uncertainties associated to the (partial differential equation) model parameters easily. The FIR model developed is able to predict accurately the global temperature change in year 2100. However, the FIR model is not easily interpretable for policy makers, due to the fact that is composed by quite a large number of rules, i.e. 56 and that the rules are pattern rules that have not a really direct interpretation in linguistic human terms. For this reasons, the LR-FIR methodology (Linguistic rules in FIR), is used to obtain a linguistic rules model that starting from the FIR model is able to capture the behavior of the system in a simple understandable manner. The set of logical rules obtained are validated by climate experts and become a useful tool for policy makers.

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